Oleh :

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1) Tentukanlah luas daerah yang dibatasi oleh kurva y = $3x^2 - 12x$ dan sumbu x!

   Jawaban :

   $3x^2 - 12x = 0$

   $3x(x - 4) = 0$

   $x = 0 \, v \, x = 4$

   $L = \int_0^4 [0 - (3x^2 - 12x)] \, dx$

   $= - [x^3 - 6x^2]_0^4$

   $= - [(4^3 - 6(4)^2)] - (0^3 - 6(0)^2]$

   $= - [(64 - 6(16)) - 0]$

   $= - [(64 - 96) - 0]$

   $= 32 \, satuan \, luas \, daerah$

2) Tentukanlah luas daerah yang dibatasi oleh kurva $y = \frac{1}{4}x^3$ dan $y = -\frac{1}{2}x^2 + 2x$!

   Jawaban:

   a. $y = \frac{1}{4}x^3$

      TPS $x \rightarrow y = 0$

      $0 = \frac{1}{4}x^3$

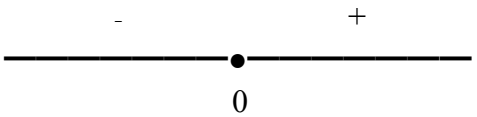
      $x_{123} = 0 \rightarrow$ (0,0), satu titik

      TPS $y \rightarrow x = 0$

      $y = \frac{1}{4}(0)^3$

      $y = 0$

      UJI TITIK

   b. $y = -\frac{1}{2}x^2 + 2x$

      TPS $x \rightarrow y = 0$

      $0 = -\frac{1}{2}x^2 + 2x$

$0 = x(-\frac{1}{2}x + 2)$

$x_1 = 0 \rightarrow (0,0)$

$x_2 = 4 \rightarrow (4,0)$

TPS $y \rightarrow x = 0$

$y = -\frac{1}{2}(0)^2 + 2(0)$

$y = 0 \rightarrow (0,0)$, satu titik

c. TP $y = \frac{1}{4}x^3$ & $y = -\frac{1}{2}x^2 + 2x$

$y = y$

$\frac{1}{4}x^3 = -\frac{1}{2}x^2 + 2x$

$\frac{1}{4}x^3 + \frac{1}{2}x^2 - 2x = 0$

$x(\frac{1}{4}x^2 + \frac{1}{2}x - 2) = 0$

——————————————— x4

$x(x^2 + 2x - 8) = 0$

$x(x + 4)(x - 2) = 0$

$x_1 = 0$

$x_2 = -4$

$x_3 = 2$

$L = [\int_0^2 (-\frac{1}{2}x^2 + 2x) - (\frac{1}{4}x^3)]\,dx + [\int_{-4}^0 (\frac{1}{4}x^3) - (-\frac{1}{2}x^2 + 2x)]\,dx$

$= [-\frac{1}{12}x^4 - \frac{1}{6}x^3 + x^2]_0^2 + [\frac{1}{12}x^4 + \frac{1}{6}x^3 - x^2]_{-4}^0$

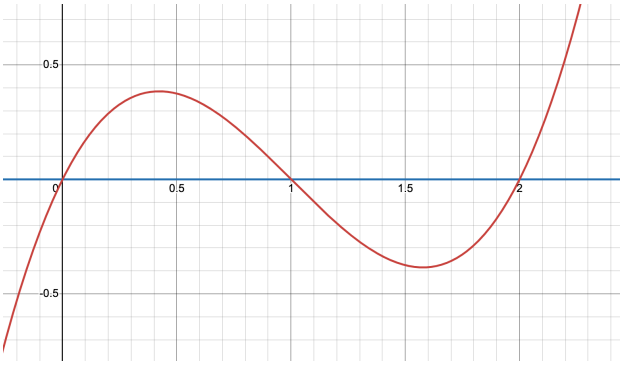
$= [-\frac{1}{12}(2)^4 - \frac{1}{6}(2)^3 + (2)^2] - [0] + [0]$

$- [\frac{1}{12}(-4)^4 + \frac{1}{6}(-4)^3 - (-4)^2]$

$= [-\frac{4}{3} - \frac{4}{3} + 4 - \frac{64}{3} + \frac{32}{3} + 16]$

$= \frac{20}{3}$ *satuan luas daerah*

3) Hitung luas daerah yang dibatasi oleh kurva $y = x^3 - 3x^2 + 2x$ dan sumbu X!



Jawaban :

Titik potong

$x^3 - 3x^2 + 2x = 0$

$x(x-2)(x-1) = 0$

$x = 0 \; v \; x = 1 \; v \; x = 2$

$$Luas = \int_0^1 [x^3 - 3x^2 + 2x - 0]dx + \int_1^2 [0 - (x^3 - 3x^2 + 2x)]dx$$

$$= \int_0^1 [x^3 - 3x^2 + 2x]dx + \int_1^2 [-x^3 + 3x^2 - 2x)]dx$$

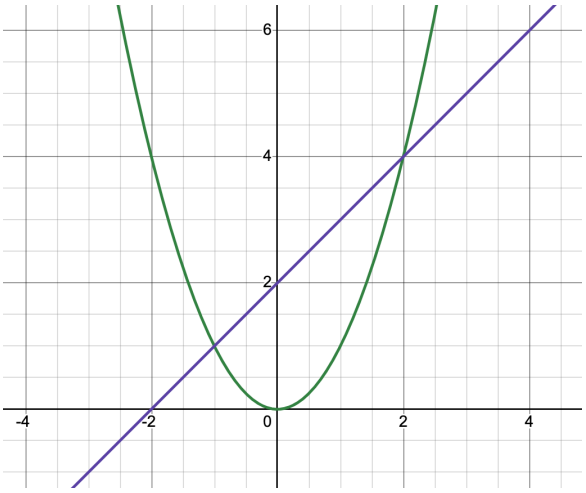
$$= [\tfrac{1}{4}x^4 - x^3 + x^2]_0^1 + [-\tfrac{1}{4}x^4 + x^3 - x^2]_1^2$$

$$= [(\tfrac{1}{4} - 1 + 1) - 0] + [(-4 + 8 - 4) - (-\tfrac{1}{4} + 1 - 1)]$$

$$= \tfrac{1}{4} + \tfrac{1}{4}$$

$$= \tfrac{1}{2} \; satuan \; luas \; daerah$$

4) Hitung luas daerah yang dibatasi oleh kurva $y = x^2$ dan sumbu $y = x + 2$!



Jawaban :

Titik potong

$x^2 = x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1 \; v \; x = 2$

$$Luas = \int_{-1}^2 [(x+2) - x^2]dx$$

$$= \int_{-1}^2 [-x^2 + x + 2]dx$$

$$= [-\tfrac{1}{3}x^3 + \tfrac{1}{2}x^2 + 2x]_{-1}^2$$

$$= (-\tfrac{8}{3} + 2 + 4) - (\tfrac{1}{3} + \tfrac{1}{2} - 2$$

$$= \left(-\frac{8}{3} + 2 + 4\right) - \left(\frac{1}{3} + \frac{1}{2} - 2\right)$$
$$= \frac{9}{2}\ satuan\ luas\ daerah$$

5) Diketahui fungsi permintaan suatu produk adalah P = 50 - 2Q dan fungsi penawarannya adalah P = 10 + 3Q. Tentukan besar surplus konsumen pada tingkat keseimbangan pasar!
Jawaban:

$$Pd = Ps$$
$$50 - 2Q = 10 + 3Q$$
$$50 - 10 = 3Q + 2Q$$
$$40 = 5Q$$
$$Qe = 8$$

$$Pe = 50 - 2(8)$$
$$Pe = 50 - 16$$
$$Pe = 34$$
$$Sc = \int_0^{Qe} [Pd - Pe]\, dQ$$
$$= \int_0^8 [(50 - 2Q) - 34]\, dQ$$
$$= \int_0^8 [16 - 2Q]\, dQ$$
$$= [16Q - Q^2]_0^8$$
$$= [16(8) - (8)^2] - 0$$
$$= [128 - 64] - 0$$
$$= 64$$

6) Diketahui fungsi permintaan suatu produk adalah P = 65 - Q² dan fungsi penawarannya adalah P = 15 + Q². Hitunglah besar surplus produsen pada kondisi keseimbangan pasar!
Jawaban:

$$Pd = Ps$$
$$65 - Q^2 = 15 + Q^2$$
$$65 - 15 = 2Q^2$$
$$Q^2 = 25$$
$$Qe = 5\ v\ Qe = -5\ (TM)$$

$$Pe = 15 - (5)^2$$

$$Pe = 15 + 25$$
$$Pe = 40$$

$$Sc = \int_0^{Qe} [Pe - Ps]\, dQ$$
$$= \int_0^5 [40 - (15 + Q^2)]\, dQ$$
$$= \int_0^5 [25 - Q^2]\, dQ$$
$$= [25Q - \tfrac{1}{3}Q^3]_0^5$$
$$= [25(5) - \tfrac{1}{3}(5)^3] - 0$$
$$= [125 - \tfrac{125}{3}] - 0$$
$$= \tfrac{125}{3}$$