Tugas Kolaborasi TIK dan Matematika Wajib

Anggota Kelompok 9:
Brandon Melano XII-C1/07
Jennifer Vanessa XII-C1/20
Jessica Leonora XII-C1/23
Naomi Ailynn XII-C1/31

1. Jika fungsi permintaan suatu barang P = 15 - 0,75Q dan tingkat harga pasar P_e = 6, maka tentukan surplus konsumen.

   Diketahui:
   P = 15 - 0,75Q
   P_e = 6
   Ditanya:
   CS (Surplus Konsumen)
   Jawab:
   $P_e = 15 - 0{,}75Q_e$
   $6 = 15 - 0{,}75Q_e$
   $6 - 15 = -0{,}75Q_e$
   $-9 = -0{,}75Q_e$
   $Q_e = \frac{-9}{-0{,}75}$
   $Q_e = 12$

   $CS = \int_0^{12} (P - P_e)\, dQ$
   $CS = \int_0^{12} ((15 - 0{,}75Q) - 6)\, dQ$
   $CS = \int_0^{12} (9 - 0{,}75Q)\, dQ$
   $CS = [9Q - \frac{3}{8} Q^2]_0^{12}$
   $CS = [9(12) - \frac{3}{8} (12)^2] - [9(0) - \frac{3}{8} (0)^2]$
   $CS = 108 - 54$
   CS = 54

2. Jika fungsi penawaran suatu barang P = 0,25Q + 5 dan tingkat harga pasar P_e = 10, maka tentukan surplus produsen.

Diketahui:

P = 0,25Q + 5

P_e = 10

Ditanya:

PS (Surplus Produsen)

Jawab:

10 = 0,25 Q_e + 5

$5 = \frac{1}{4} Q_e$

Q_e = 20

$$PS = \int_0^{20} (P_e - P)\, dQ$$

$$PS = \int_0^{20} (10 - (\frac{1}{4} Q + 5))\, dQ$$

$$PS = \int_0^{20} (5 - \frac{1}{4} Q)\, dQ$$

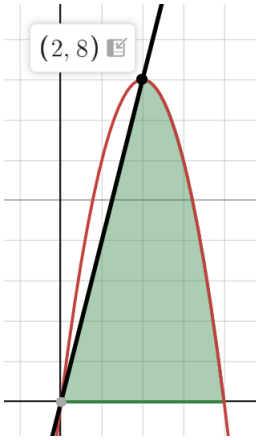
$$PS = [5Q - \frac{1}{8} Q^2]_0^{20}$$

$$PS = [5(20) - \frac{1}{8} (20)^2] - [5(0) - \frac{1}{8} (0)^2]$$

PS = [100 - 50] - 0

PS = 50

3. Find the area of the shaded region.



Diketahui:

- Parabola dengan titik puncak (h, k) = (2, 8) dan melalui titik (0, 0)
- Garis melalui titik (0, 0) dan (2, 8)

Ditanya:

L_{total} (Luas daerah terarsir)

Jawab:

<u>Persamaan Parabola:</u>

$y = a(x - h)^2 + k$

$0 = a(0 - 2)^2 + 8$

$0 = 4a + 8$

$-8 = 4a$

$a = -2$

$y = -2(x - 2)^2 + 8$

$y = -2(x^2 - 4x + 4) + 8$

$y = -2x^2 + 8x$

Persamaan Garis:

$$\frac{(y-y_1)}{(y_2-y_1)} = \frac{(x-x_1)}{(x_2-x_1)}$$

$$\frac{y-8}{0-8} = \frac{x-2}{0-2}$$

$-2y + 16 = -8x + 16$

$-2y = -8x$

$y = 4x$

Luas Daerah 1 (L_1):

$$L_1 = \int_0^2 (4x - 0)\, dx$$

$$L_1 = \int_0^2 4x\, dx$$

$$L_1 = [2x^2]_0^2$$

$L_1 = 2(2)^2 - 2(0)^2$

$L_1 = 8$ satuan luas

Luas Daerah 2 (L_2):

$$L_2 = \int_2^4 [(-2x^2 + 8x) - 0]\, dx$$

$$L_2 = \int_2^4 (-2x^2 + 8x)\, dx$$

$$L_2 = [-\frac{2}{3}x^3 + 4x^2]_0^2$$

$$L_2 = [-\frac{2}{3}(4)^3 + 4(4)^2] - [-\frac{2}{3}(2)^3 + 4(2)^2]$$

$$L_2 = [-\frac{128}{3} + 64] - [-\frac{16}{3} + 16]$$

$$L_2 = [\frac{64}{3}] - [\frac{32}{3}]$$

$L_2 = \frac{32}{3}$ satuan luas

Luas Total (L_{total}):

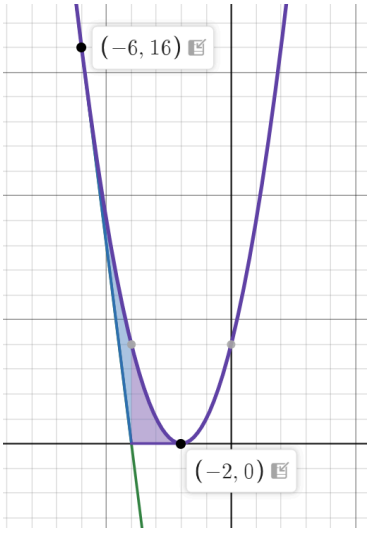
$L_{total} = L_1 + L_2$

$L_{total} = 8 + 3$

$L_{total} = (24 + 32)/3$

$L_{total} = \frac{56}{3}$ satuan luas

4. Dari gambar disamping hitunglah luas daerah yang diarsir jika garis g adalah garis singgung parabola di titik (-6, 16).



Diketahui:

- Parabola dengan titik puncak (-2, 0) melalui titik (-6, 16)
- Garis g menyinggung parabola di titik (-6, 16)

Ditanya:

L_{total} (Luas daerah terarsir)

Jawab:

Persamaan Parabola:

$y = a(x - h)^2 + k$

$16 = a(-6 + 2)^2 + 0$

$16 = a(-4)^2$

$16 = 16a$

$a = 1$

$y = (x + 2)^2 + 0$

$y = x^2 + 4x + 4$

Persamaan Garis Singgung g:

$$\frac{(y-y_1)}{(y_2-y_1)} = \frac{(x-x_1)}{(x_2-x_1)}$$

$$\frac{y-16}{0-16} = \frac{x+6}{-4+6}$$

$2y - 32 = -16x - 96$

$2y = -16x - 64$

$y = -8x - 32$

Luas Daerah 1 (L_1):

$$L_1 = \int_{-6}^{-4} [(x^2 + 4x + 4) - (-8x - 32)]\, dx$$

$$L_1 = \int_{-6}^{-4} (x^2 + 12x + 36)\, dx$$

$L_1 = [-\frac{1}{3}x^3 + 6x^2 + 36x]_{-6}^{-4}$

$L_1 = [\frac{1}{3}(-4)^3 + 6(-4)^2 + 36(-4)] - [\frac{1}{3}(-6)^3 + 6(-6)^2 + 36(-6)]$

$L_1 = [-\frac{64}{3} + 96 - 144] - [-\frac{216}{3} + 216 - 216]$

$L_1 = [-\frac{64}{3} - 48] - [-72]$

$L_1 = [-\frac{208}{3}] + \frac{216}{3}$

$L_1 = \frac{8}{3}$ satuan luas

Luas Daerah 2 (L_2):

$L_2 = \int_{-4}^{-2} [(x^2 + 4x + 4) - 0]\, dx$

$L_2 = [\frac{1}{3}x^3 + 2x^2 + 4x]_{-4}^{-2}$

$L_2 = [\frac{1}{3}(-2)^3 + 2(-2)^2 + 4(-2)] - [\frac{1}{3}(-4)^3 + 2(-4)^2 + 4(-4)]$

$L_2 = [-\frac{8}{3} + 8 - 8] - [-\frac{64}{3} + 32 - 16]$

$L_2 = [-\frac{8}{3}] - [-\frac{64}{3} + 16]$

$L_2 = [-\frac{8}{3}] - [-\frac{16}{3}]$

$L_2 = -\frac{8}{3} + \frac{16}{3}$

$L_2 = \frac{8}{3}$ satuan luas

Luas Total (L_{total}):

$L_{total} = L_1 + L_2$

$L_{total} = \frac{8}{3} + \frac{8}{3}$

$L_{total} = \frac{16}{3}$ satuan luas

5. Hitung luas daerah yang dibatasi oleh $x + 3y^2 = 12$, $y = 3$, $3y - x + 6 = 0$.

   Diketahui:

   - $x = 12 - 3y^2$
   - $x = 3y + 6$
   - Batas atas $y = 3$

   Ditanya:

   L (Luas daerah)

Jawab:

Titik potong kurva:

$3y + 6 = 12 - 3y^2$

$3y^2 + 3y - 6 = 0$

$y^2 + y - 2 = 0$

$(y + 2)(y - 1) = 0$

$y = -2$ atau $y = 1$

Menghitung Luas (L):

$L = \int_1^3 [(3y + 6) - (12 - 3y^2)]\, dy$

$L = \int_1^3 (3y - 6 + 3y^2)\, dy$

$L = [\frac{3}{2}y^2 - 6y + y^3]_1^3$

$L = [\frac{3}{2}(3)^2 - 6(3) + (3)^3] - [\frac{3}{2}(1)^2 - 6(1) + (1)^3]$

$L = [\frac{27}{2} - 18 + 27] - [\frac{3}{2} - 6 + 1]$

$L = [\frac{27}{2} + 9] - [\frac{3}{2} - 5]$

$L = [\frac{45}{2}] - [-\frac{7}{2}]$

$L = \frac{52}{2}$

L = 26 satuan luas

6. Hitung luas daerah yang dibatasi oleh kurva $y = x^2 + 1$, $y = -2x$, dan sumbu Y.

Diketahui:

- $y = x^2 + 1$
- $y = -2x$
- Sumbu Y $\Longrightarrow x = 0$

Ditanya:

L (Luas daerah)

Jawab:

Titik potong kurva:

$-2x = x^2 + 1$

$0 = x^2 + 2x + 1$

$0 = (x + 1)(x + 1)$

$x = -1$

Menghitung Luas (L):

$L = \int_{-1}^{0} [(x^2 + 1) - (-2x)]\, dx$

$L = \int_{-1}^{0} (x^2 + 2x + 1)\, dx$

$L = [\frac{1}{3}x^3 + x^2 + x]_{-1}^{0}$

$L = [\frac{1}{3}(0)^3 + (0)^2 + (0)] - [\frac{1}{3}(-1)^3 + (-1)^2 + (-1)]$

$L = [0] - [-\frac{1}{3} + 1 - 1]$

$L = [0] - [-\frac{1}{3}]$

L = $\frac{1}{3}$ satuan luas