Kelompok 5 XII-C1

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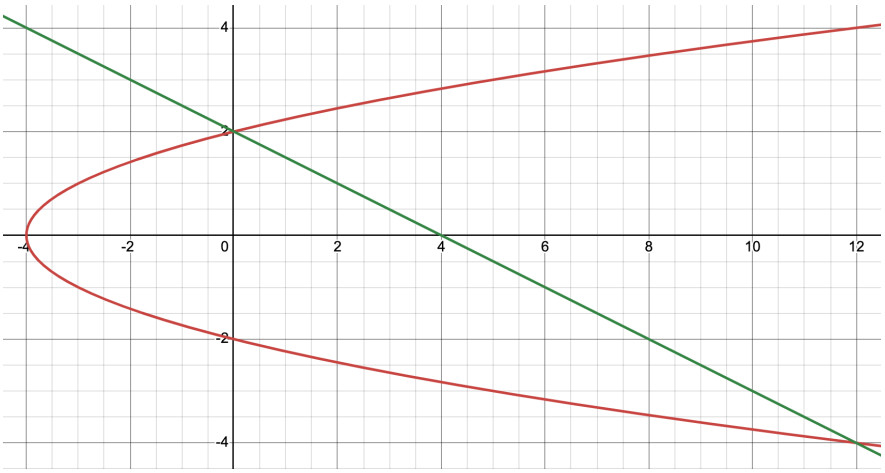
Soal 1

Find the region area which is bordered by $x = y^2 - 4$ and $y = 2 - \frac{1}{2}x$!

Pembahasan:

Gambarkan terlebih dahulu!



$y = 2 - \frac{1}{2}x \rightarrow 2y = 4 - x \rightarrow x = 4 - 2y$

Mencari titik potong:

$$x_1 = x_2$$

$$y^2 - 4 = 4 - 2y$$

$$y^2 + 2y - 8 = 0$$

$$(y + 4)(y - 2) = 0$$

$$y = -4 \vee y = 2$$

Mencari luas daerah yang dibatasi:

$$L = \int_{-4}^{2} [(4 - 2y) - (y^2 - 4)]dy$$

$$L = \int_{-4}^{2} (-y^2 - 2y + 8)dy$$

$$L = \left[-\frac{1}{3}y^3 - y^2 + 8y\right]_{-4}^{2}$$

$$L = \left[-\frac{1}{3}y^3 - y^2 + 8y\right]_{-4}^{2}$$

$$L = \left(-\frac{1}{3} \cdot 2^3 - 2^2 + 8 \cdot 2\right) - \left(-\frac{1}{3} \cdot (-4)^3 - (-4)^2 + 8 \cdot (-4)\right)$$

$$L = \left(-\frac{8}{3} - 4 + 16\right) - \left(\frac{64}{3} - 16 - 32\right)$$

$$L = \left(-\frac{8}{3} + 12\right) - \left(\frac{64}{3} - 48\right)$$

$$L = \left(\frac{-8 + 36}{3}\right) - \left(\frac{64 - 144}{3}\right)$$

$$L = \frac{28}{3} - \left(\frac{-80}{3}\right)$$

$$L = \frac{28}{3} + \frac{80}{3}$$

$$L = \frac{108}{3}$$

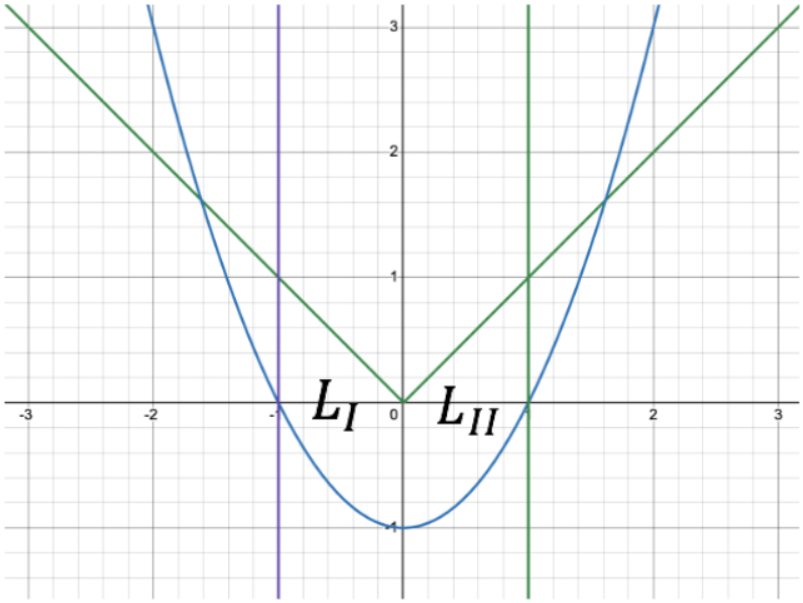
$$\therefore L = 36 \; satuan \; luas$$

Soal 2

Find the region area which is bordered by $y = |x|$, $y = x^2 - 1$, $x = 1$, $x = -1$!

Pembahasan:

Gambarkan terlebih dahulu!



$y = |x|$ dapat dibagi menjadi $y = -x$ untuk garis di sebelah kiri sumbu$-Y$ dan $y = x$ untuk garis di terletak di sebelah kanan sumbu$-Y$.

Menghitung luas L_I :

$$L_I = \int_{-1}^{0} [(-x) - (x^2 - 1)]dx$$

$$L_I = \int_{-1}^{0} (-x^2 - x + 1)dx$$

$$L_I = \left[-\frac{1}{3}x^3 - \frac{1}{2}x^2 + x\right]_{-1}^{0}$$

$$L_I = 0 - \left[-\frac{1}{3}\cdot(-1)^3 - \frac{1}{2}\cdot(-1)^2 + (-1)\right]$$

$$L_I = -\left(\frac{1}{3} - \frac{1}{2} - 1\right)$$

$$L_I = -\left(\frac{2}{6} - \frac{3}{6} - \frac{6}{6}\right)$$

$$L_I = -\left(-\frac{7}{6}\right)$$

$$L_I = \frac{7}{6}\ satuan\ luas$$

Menghitung luas L_{II} :

$$L_{II} = \int_0^1 [(x) - (x^2 - 1)]dx$$

$$L_{II} = \int_0^1 (-x^2 + x + 1)dx$$

$$L_{II} = \left[-\frac{1}{3}x^3 + \frac{1}{2}x^2 + x\right]_0^1$$

$$L_{II} = \left[-\frac{1}{3}\cdot 1^3 + \frac{1}{2}\cdot 1^2 + 1\right] - 0$$

$$L_{II} = -\frac{1}{3} + \frac{1}{2} + 1$$

$$L_{II} = -\frac{2}{6} + \frac{3}{6} + \frac{6}{6}$$

$$L_{II} = \frac{7}{6}$$

$$L_{II} = \frac{7}{6}\ satuan\ luas$$

Mencari luas total:

$$L_{total} = L_I + L_{II}$$

$$L_{total} = \frac{7}{6} + \frac{7}{6}$$

$$L_{total} = \frac{14}{6}$$

$$L_{total} = \frac{7}{3}$$

$$\therefore L_{total} = 2\frac{1}{3}\ satuan\ luas$$

Soal 3

A company manufactures dan distributes organic fertilizer to agricultural suppliers. As production increases, the company experiences higher production costs due to the increasing demand for raw materials, labor, and transportation.

An economic analyst establishes the following supply function:

$$P_s(Q) = 3Q^2 + 8Q + 20$$

Where:

- $P_s(Q)$ = minimum selling price per production unit (in thousand rupiahs).
- Q = quantity supplied (in production units).

During the agricultural season, the market price of organic fertilizer rises to Rp1.380.000,00 per production unit. Based on the information above, calculate the Producer Surplus (P_s) obtained by the company using the definite integral method. Show your calculations step by step!

Solution:

Given:

- $P_s = 3Q^2 + 8Q + 20$
- $P_e = 1380$

First, determine the equilibrium quantity:

$$1380 = 3Q^2 + 8Q + 20$$
$$3Q^2 + 8Q - 1360 = 0$$
$$(Q - 20)(3Q + 68) = 0$$

Since quantity cannot be negative:

$$Q_e = 20$$

Calculate Producer Surplus:

$$P_s = P_e Q_e - \int_0^{Q_e} P_s(Q)dQ$$
$$P_s = (1380)(20) - \int_0^{20} (3Q^2 + 8Q + 20)dQ$$
$$P_s = 27600 - [Q^3 + 4Q^2 + 20Q]_0^{20}$$
$$P_s = 27600 - (8000 + 1600 + 400)$$

$$P_s = 27600 - 10000$$
$$P_s = 17600$$

So, the producer surplus earned by the organic fertilizer manufacturing company is $Rp17.600.000{,}00$. This represents the difference between the total revenue received by the company at the market price and the minimum total amount required by the producer to supply the goods.

Soal 4

Fungsi penawaran suatu barang dirumuskan $P = Q^2 - 4$ dan fungsi permintaan $P = -Q^2 + 14$. Hitung besarnya surplus produsen!

Pembahasan:

Equlibrium

$$P = P$$

$$Q_e{}^2 - 4 = -Q_e^2 + 14$$

$$2Q_e{}^2 = 18$$

$$Q_e{}^2 = 9$$

$$Q_e = 3 \vee Q_e = -3(TM)$$

Mencari P_e :

$$P_e = Q_e^2 - 4$$

$$P_e = 3^2 - 4$$

$$P_e = 5$$

Mencari P_s :

$$P_s = \int_0^{Q_e} (P_e - P)dQ$$

$$P_s = \int_0^3 [5 - (Q^2 - 4)]dQ$$

$$P_s = \int_0^3 (-Q^2 + 9)dQ$$

$$P_s = \left[-\frac{1}{3}Q^3 + 9Q\right]_0^3$$

$$P_s = \left(-\frac{1}{3} \cdot 3^3 + 9 \cdot 3\right) - 0$$

$$\therefore P_s = 18$$

Soal 5

Diketahui dua kurva:

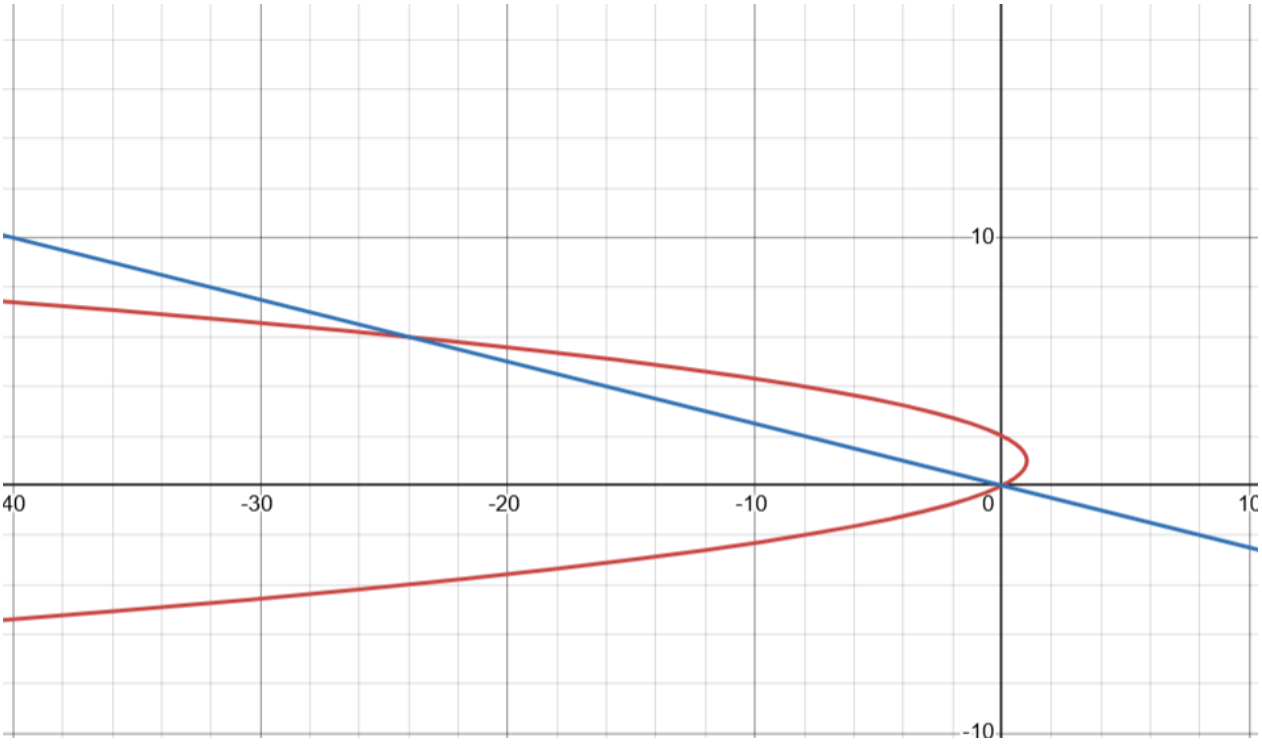
$$y = -\frac{1}{4}x$$

$$x = -(y-1)^2 + 1$$

Tentukan luas daerah yang dibatasi oleh kedua kurva tersebut.

Pembahasan:

Gambarkan terlebih dahulu!



Mengubah persamaan dalam bentuk $x =$:

$$y = -\frac{1}{4}x$$

$$x = -4y$$

Titik potong $x = -4y$ dan $x = -(y-1)^2 + 1$:

$$x_1 = x_2$$

$$-4y = -(y-1)^2 + 1$$

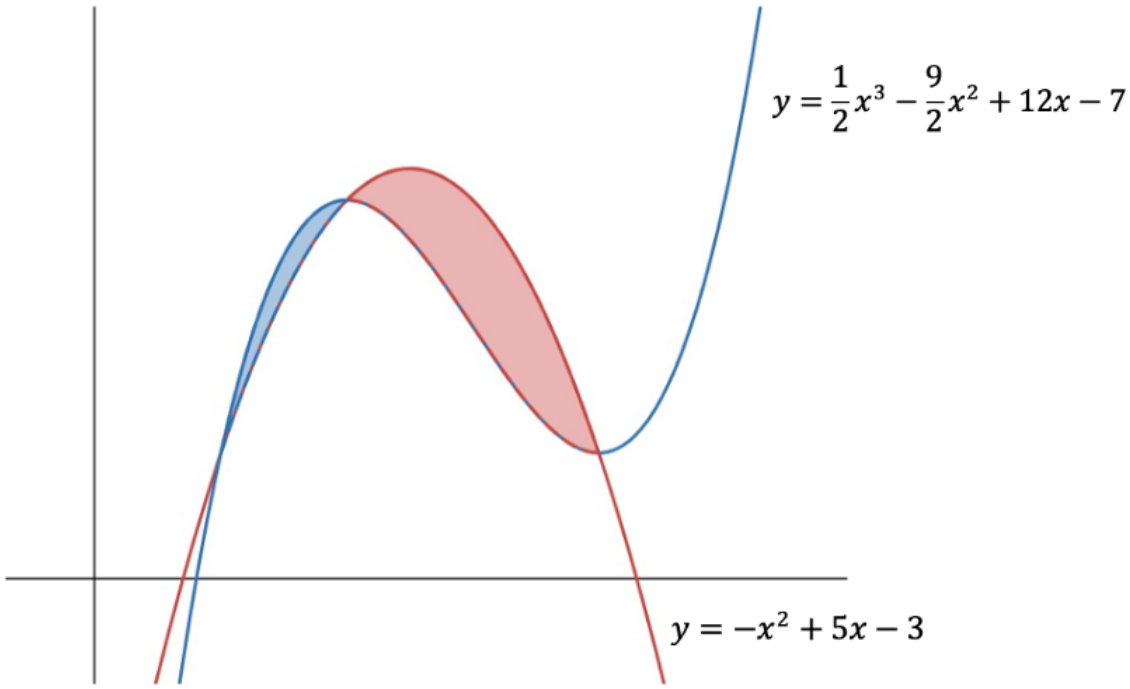
$$-4y = -y^2 + 2y - 1 + 1$$
$$0 = y^2 - 2y - 4y + 1 - 1$$
$$0 = y^2 - 6y$$
$$0 = y(y - 6)$$
$$y = 0 \vee y = 6$$

Menghitung luas yang dibatasi:

$$L = \int_0^6 [(-(y-1)^2 + 1) - (-4y)]dy$$
$$L = \int_0^6 (-(y^2 - 2y + 1) + 1 + 4y)dy$$
$$L = \int_0^6 (-y^2 + 2y - 1 + 1 + 4y)dy$$
$$L = \int_0^6 (-y^2 + 6y)dy$$
$$L = \left[-\frac{y^3}{3} + 3y^2\right]_0^6$$
$$L = \left(\frac{6^3}{3} + 3 \cdot 6^2\right) - (0)$$
$$L = -\frac{216}{3} + 3(36)$$
$$L = -72 + 108$$
$$\therefore L = 36\ satuan\ luas$$

Soal 6

Perhatikan grafik di bawah ini. Hitunglah luas daerah yang dibatasi oleh kedua kurva tersebut!



Pembahasan:

Mencari titik potong:

$$y_1 = y_2$$

$$\frac{1}{2}x^3 - \frac{9}{2}x^2 + 12x - 7 = -x^2 + 5x - 3$$

$$\frac{1}{2}x^3 - \frac{9}{2}x^2 + x^2 + 12x - 5x - 7 + 3 = 0$$

$$\frac{1}{2}x^3 - \frac{7}{2}x^2 + 7x - 4 = 0$$

$$x^3 - 7x^2 + 14x - 8 = 0$$

Coba substitusi $x = 1$:

$$1^3 - 7 \cdot 1^2 + 14 \cdot 1 - 8 = 1 - 7 + 14 - 8 = 0$$

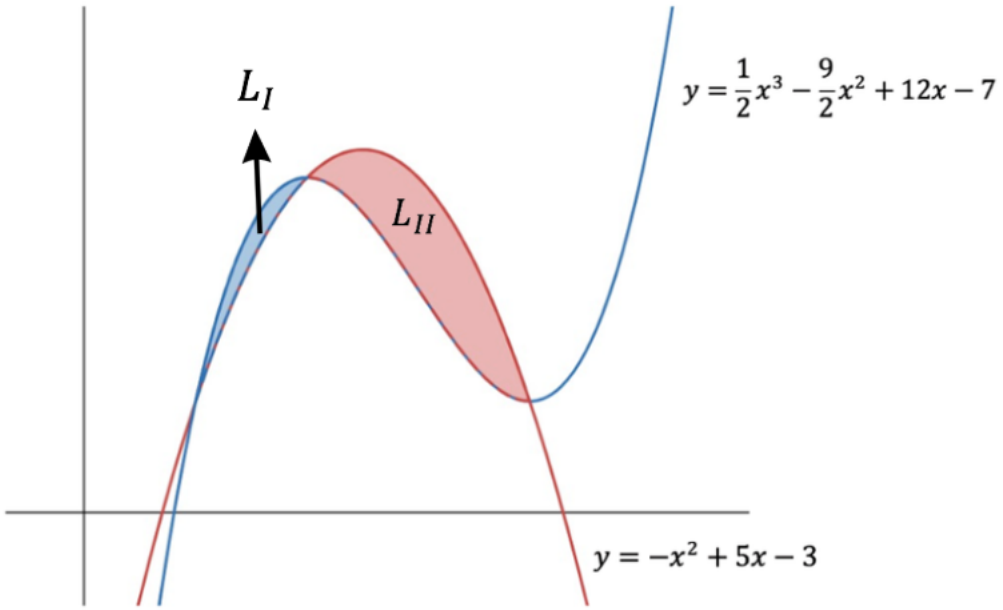
Karena hasilnya 0, maka $(x - 1)$ adalah salah satu faktornya.
Menggunakan pembagian bersusun atau Horner, kita akan mendapatkan:

$$x^3 - 7x^2 + 14x - 8 = (x - 1)(x^2 - 6x + 8)$$

$$(x - 1)(x^2 - 6x + 8) = 0$$

$$(x-1)(x-4)(x-2)=0$$

$$x=1 \vee x=4 \vee x=2$$



Untuk mencari luas total daerah yang dibatasi, maka kita harus terlebih dahulu mencari L_I dan L_{II} :

$$L_{total}=L_I+L_{II}$$

Mencari L_I:

$$L_I=\int_1^2\left[\left(\frac{1}{2}x^3-\frac{9}{2}x^2+12x-7\right)-(-x^2+5x-3)\right]dx$$

$$L_I=\int_1^2\left(\frac{1}{2}x^3-\frac{9}{2}x^2+12x-7+x^2-5x+3\right)dx$$

$$L_I=\int_1^2\left(\frac{1}{2}x^3-\frac{7}{2}x^2+7x-4\right)dx$$

$$L_I=\left[\frac{1}{8}x^4-\frac{7}{6}x^3+\frac{7}{2}x^2-4x\right]_1^2$$

$$L_I=\left(\frac{1}{8}\cdot 2^4-\frac{7}{6}\cdot 2^3+\frac{7}{2}\cdot 2^2-4\cdot 2\right)-\left(\frac{1}{8}\cdot 1^4-\frac{7}{6}\cdot 1^3+\frac{7}{2}\cdot 1^2-4\cdot 1\right)$$

$$L_I=\left(2-\frac{28}{3}+14-8\right)-\left(\frac{1}{8}-\frac{7}{6}+\frac{7}{2}-4\right)$$

$$L_I=\left(8-\frac{28}{3}\right)-\left(\frac{3-28+84-96}{24}\right)$$

$$L_I = \left(\frac{24-28}{3}\right) - \left(-\frac{37}{24}\right)$$

$$L_I = -\frac{4}{3} + \frac{37}{24}$$

$$L_I = \frac{-32+37}{24}$$

$$L_I = \frac{5}{24}\ satuan\ luas$$

Mencari L_{II}:

$$L_{II} = \int_2^4 \left[(-x^2+5x-3) - \left(\frac{1}{2}x^3 - \frac{9}{2}x^2 + 12x - 7\right)\right] dx$$

$$L_{II} = \int_2^4 \left(-\frac{1}{2}x^3 + \frac{9}{2}x^2 - 12x + 7 - x^2 + 5x - 3\right) dx$$

$$L_{II} = \int_2^4 \left(-\frac{1}{2}x^3 + \frac{7}{2}x^2 - 7x + 4\right) dx$$

$$L_{II} = \left[-\frac{1}{8}x^4 + \frac{7}{6}x^3 - \frac{7}{2}x^2 + 4x\right]_2^4$$

$$L_{II} = \left(-\frac{1}{8}\cdot 4^4 + \frac{7}{6}\cdot 4^3 - \frac{7}{2}\cdot 4^2 + 4\cdot 4\right) - (-\frac{1}{8}\cdot 2^4 + \frac{7}{6}\cdot 2^3 - \frac{7}{2}\cdot 2^2 + 4\cdot 2)$$

$$L_{II} = \left(-32 + \frac{224}{3} - 56 + 16\right) - (-2 + \frac{28}{3} - 14 + 8)$$

$$L_{II} = \left(-72 + \frac{224}{3}\right) - (-8 + \frac{28}{3})$$

$$L_{II} = \left(\frac{-216+224}{3}\right) - (\frac{-24+28}{3})$$

$$L_{II} = \frac{8}{3} - \frac{4}{3}$$

$$L_{II} = \frac{4}{3}\ satuan\ luas$$

Mencari luas total:

$$L_{total} = L_I + L_{II}$$

$$L_{total} = \frac{5}{24} + \frac{4}{3}$$

$$L_{total} = \frac{5 + 32}{24}$$

$$L_{total} = \frac{37}{24}$$

$$\therefore L_{total} = 1\frac{13}{24}\ satuan\ luas$$